Hierarchical Optimization Time Integration for CFL-rate MPM Stepping

1 Benchmark Summary Table

For performance and convergence comparison, we put timing and iteration results in the following two tables. *avg time* measures average absolute cost (seconds) per playback frame, *total* measures the HOT speedup factor of the wall clock time for the entire rendered animation sequence, *max* records the maximum speedup factor HOT achieved on a simulated (and rendered) at 24Hz frame, *avg iter* (or *iter*) measures the average number of Newton or quasi-Newton outer iterations (per method) required per frame to achieve the requested accuracy. Each example is run for all methods on the same machine. Machines employed per example: *Twist*, *Chain* and *Wheel*: Intel Core i7-7700K; all other examples are run on an Intel Core i7-8700K. Both machines has 64GB memory. Cat Young's modulus values are $^\dagger 10^6$ and $^\ddagger 10^9$ respectively. * indicates that the examples could not finish in reasonable time, and was manually terminated.

Table 1: **Newton's Method Timings:** Here we summarize statistics across all benchmark examples using Newton's methods (including the previous state-of-the-art Gast15 [1] in comparison with HOT. Here, Gast15 method consistently adopts 1e-3 as the outer tolerance for all examples, which is the maximum that guarantees artifact-free results.

Example	HOT		Gast15(MF)			PN-PCG			PN-PCG(MF)			PN-MGPCG		
	avg time	avg iter	avg time	total	iter	total	max	iter	total	max	iter	total	max	iter
Twist	77.73	13.49	*2308.70	*29.70×	*19.33	4.65×	8.17×	11.14	4.73×	9.57×	11.14	6.79×	9.85×	5.42
Boxes	129.81	5.76	*10142.33	*78.13×	*12.14	3.59×	9.29×	7.21	3.73×	9.19×	7.21	3.57×	7.91×	3.94
Donut	121.19	27.76	*1150.41	*9.49×	*15.68	1.98×	7.61×	9.07	1.98×	9.39×	9.07	10.67×	17.97×	4.68
†ArmaCat	32.55	6.22	62.78	1.93×	8.60	3.41×	4.53×	7.03	1.22×	1.79×	7.03	3.21×	3.87×	4.69
‡ArmaCat	36.61	8.72	324.77	8.87×	13.94	4.19×	6.28×	8.40	2.02×	3.78×	8.40	3.42×	3.43×	5.38
Chain	98.78	5.55	*766.47	*7.76×	*9.84	5.79×	11.99×	6.04	1.98×	6.85×	6.04	4.02×	8.69×	3.42
Boards	105.99	3.72	296.43	2.80×	2.74	2.95×	5.77×	3.11	1.73×	7.39×	3.11	2.51×	4.76×	2.402
Wheel	44.38	8.56	*39447.37	*888.85×	* 54.5	4.64×	5.93×	8.42	5.76×	6.74×	8.42	3.58×	4.88×	5.96
Faceless	3.49	6.44	2.84	0.81×	2.09	2.06×	5.74×	4.49	1.68×	7.05×	4.49	2.25×	6.42×	3.81
Sauce	13.11	4.54	10.42	0.79×	3.21	2.22×	5.77×	4.93	1.05×	2.69×	4.93	2.26×	2.82×	3.18

Table 2: **HOT Timing Comparisons:** Here we summarize statistics across all benchmark examples and methods that partly resemble our HOT. Compared to HOT, both LBFGS-GMG and LBFGS-H use LBFGS as the quasi-Newton solver but with different initializers, i.e. baseline particle quadrature multigrid for LBFGS-GMG and inexact PCG for LBFGS-H. PN-MGPCG adopts the same multigrid formulation from HOT yet a different nonlinear optimization method. HOT-quadratic is the derivation of HOT whose multigrid is built according to quadratic kernel rather than linear kernel. As a result, all these alternatives are much less efficient than HOT in general.

Example	HOT		HOT-quadratic			LBFGS-GMG		LBFGS-H			PN-MGPCG		
	avg time	avg iter	total	max	iter	total	iter	total	max	iter	total	max	iter
Twist	77.73	13.49	7.10×	86.42×	51.24	*186.93×	*1234.94	4.12×	9.53×	20.45	6.79×	9.85×	5.42
Boxes	129.81	5.76	2.54×	4.60×	9.61	*61.41×	*296.56	2.39×	8.84×	6.78	3.57×	7.91×	3.94
Donut	121.19	27.76	2.18×	4.59×	32.81	*85.38×	*1182.52	4.79×	2.63×	16.42	10.67×	17.97×	4.68
†ArmaCat	32.55	6.22	2.01×	2.09×	6.17	2.93×	18.70	0.94×	1.72×	8.09	3.21×	3.87×	4.69
‡ArmaCat	36.61	8.72	1.94×	3.18×	8.67	*201.56×	*709.05	1.37×	2.45×	8.95	3.42×	3.43×	5.38
Chain	98.78	5.55	2.91×	5.77×	4.54	*7.59×	*166.57	1.92×	5.83×	6.26	4.02×	8.69×	3.42
Boards	105.99	3.72	2.83×	4.09×	3.56	4.98×	39.87	2.01×	5.13×	6.252	2.51×	4.76×	2.402
Wheel	44.38	8.56	2.27×	2.49×	7.77	*2403.47×	*5817	*51.62×	*217.75×	*16.36	3.58×	4.88×	5.96
Faceless	3.49	6.44	1.80×	2.20×	6.56	6.12×	9.64	1.03×	1.31×	9.19	2.25×	6.42×	3.81
Sauce	13.11	4.54	1.97×	2.82×	4.56	2.86×	6.13	0.92×	5.45×	7.76	2.26×	2.82×	3.18

Figure 1: **Artifacts**. Various scales of explosions can be observed among *twist*, *boxes*, *donut*, and †*armacat(*1e6*)*. Artificial softening occurs in ‡*armacat(*1e9*)*, *boards*, *faceless* and *sauce*. In *chain*, rings in the middle are not pulled from each other under forces from both two sides.

2 Gast15 Failed Cases

In this section, we demonstrate all failed results (Figure 1) generated from the previous state-of-the-art Gast15 [1] using the same tolerance 10^2. These models exhibit obvious artifacts of all kinds due to the inappropriate tolerance setting in each example except for wheel. The largest tolerance that produce artifact-free results varies across examples and this inconsistency brings significant inconvenience to the setup of a new simulation, even worse for cases where material properties change throughout the simulation.

References

[1] T. Gast, C. Schroeder, A. Stomakhin, C. Jiang, and J. Teran. Optimization integrator for large time steps. *IEEE Trans Vis Comp Graph*, 21(10):1103–1115, 2015.